Time as a Third Dimension in the One Period Neoclassical Monopoly Pricing Model

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The standard Neoclassical firm chooses price and quantity to maximize profit within a unit period whose length is arbitrarily chosen by the theoretician. It is daily, weekly or monthly profit that is maximized, and, in the absence of a specific choice of length, the unit period can be defined as an indivisible instant. It is a period that cannot be subdivided in a meaningful way from the point of view of the analysis. Thus, if daily variations in demand and costs are meaningful, while those that are observed from one minute to the next are not, then the Neoclassical firm is viewed as maximizing daily rather than per minute profit. Within that day, a single price is charged to generate the revenues that will cover the costs associated to the total volume of output sold over that period. One can define the unit period as that time frame within which profit is defined. For the Neoclassical theoretician, its length is not a strategic decision variable. Casual empiricism suggests otherwise.

In the real world of manufacturing, firms in the United States are observed to hold selling prices constant for several months or even years, and to hold inventory levels that averaged 1.86 months between 1983 and 1984. Thus, at any point in time, the cost of about two months worth of sales has actually been incurred by the average company. Consequently, an essential ingredient of the profit maximization equation – namely the cost of goods sold – is associated to a non-arbitrary time frame – namely the average 1.86 months inventory-to-sales ratio. Moreover, prices remain sticky for periods that extend beyond the inventory-to-sales ratio (Cecchetti, 1985). Thus, the inventory-to-sales ratio is a time frame within which costs and revenues can be matched to define profit. It qualifies, therefore, as a unit period but differs in one essential respect from its Neoclassical incarnation: the inventory-to-sales ratio is one of the firm’s decision variables.

From a formal point of view, the standard Neoclassical firm, when faced with a set of three variables price, p, quantity, q and time, t, chooses to fix the third and examines the combinations of p and q that will maximize profit given demand within the unit period of fixed length t = t*. The fixing of one of the three variables p, q, and t is legitimate since it ensures that the relationship between the other two becomes determinate. If the time frame is fixed, there is an unambiguous answer to the question of how many units can be sold at each price. But a firm that holds inventory is constraining variable q, not variable t. A firm that sells on order is constrained by the price at which it has taken orders, and this price will be associated to the time it will take to fill those orders. Interestingly, disequilibrium theory developed, precisely, out of a recognition that price or quantity constraints were the rule rather than the exception in modern economies (see for example Barro and Grossman, 1971 or Benassy, 1975). But disequilibrium theory ignored the third dimension of the profit equation: time t. Thus, the implications of quantity constraints were examined in terms of spillovers taking place within the unit period, while the effect of price on the sales time of output was ignored. Firms practising fixed prices were analysed without introducing the idea that prices can be held constant for an endogenously determined length of time. In brief, one dimension of the problem was left out.

Adding a time dimension to the standard one period monopoly model of the firm yields interesting results, and they will be the focus of the final part of this essay. To set up the model, first choose an accounting unit for time. It can be the hour or the working day, the week or the month. Let f be fixed costs incurred during each accounting unit of time, and e(q) be variable costs that are assumed to vary only with output. Given selling price p, the quantity demanded and sold over t periods of time is q(p, t). By identity, profit per unit of time r satisfies:

\[ p \cdot q(p, t) - c(q(p, t)) - (r + f) \cdot t = 0 \]

so that:

\[ r(p, t) = \frac{p \cdot q(p, t) - c(q(p, t))}{t} - f \]
The three variables p, q and t are not independent since t is the time it takes to sell output q at price p, and p is the price at which q units are sold if a period of time of length t is allowed to elapse. Fixing one of the variables determines a one-to-one relationship between the other two. Thus the fixing of t at \( t^* \) determines the standard demand relationship q = q(p, t*), while the fixing of p will determine the evolution of cumulative sales over time. If q is fixed at some level \( q^* \), then the function t = t(p, q*) yields the sales time of output q* at all possible selling prices. Consequently, per unit time profit r can be written as a function of any two of the variables p, q and t. Thus:

\[
  r = r(p, q) = \frac{p \cdot q - c(q)}{t(p, q)} - f
\]

The quantity or price constraints of disequilibrium theory can be treated in the same way as Neoclassical equilibrium theory's time constraint. The quantity constraint can, for instance, be interpreted as the holding by a monopolist of a level of inventory \( q^* \) which translates into an incurred cost constraint \( c(q^*) \). (Interpreting \( q^* \) as an inventory level justifies making variable costs \( c(q^*) \) independent from t. In this simplified model, storage costs are assumed to be fixed costs of warehouse maintenance, and therefore included in f). The time it will take to sell that inventory \( t(p, q^* \) is also the time it will take to generate the revenues that are to be set against the cost of goods and the fixed costs accumulated with sales time t(p, q*). The maximizing monopolist then solves:

\[
  \max_p r(p, q^*) = \frac{p \cdot q^* - c(q^*)}{t(p, q^*)} - f
\]

First order condition \( \frac{\partial r}{\partial p} = 0 \) yields a formula for markup over the average cost of goods sold:

\[
  \frac{p - AC}{p} = \frac{1}{\varepsilon(p, q^*)}
\]

where \( \varepsilon = (\partial q/\partial t) \cdot (t/q) \) is the price elasticity of the sales time of inventory. This formula for price has been shown to be compatible with the markup and inventory data of a selection of countries and industries (Langlois 1989, 1990a, 1990b). This optimization assumes price to be held constant throughout the endogenously determined period of length t. Thus a price constraint comes into the picture.

Setting free the output variable, let us examine the monopolist whose objective would be to decide how long fixed price \( p = p^* \) should be maintained to maximize per unit time profit. The objective function reads:

\[
  \max_q r(p^*, q) = \frac{p^* \cdot q - c(q)}{t(p^*, q)} - f
\]

The first order condition \( \partial r/\partial q = 0 \) yields the following relationship:

\[
  \frac{(p^* - AC)/(p^* - MC)}{p^* - MC} = \xi(p^*, q)
\]

where MC is marginal cost and \( \xi = (\partial q/\partial t) \cdot (t/q) \) is the percentage change in cumulative sales that would result from a 1 percent change in the time allowed for sales to take place. If \( p^* \) is maximizing for the quantity constrained problem, then \( (p^* - AC)/p^* = (1/\varepsilon(p^*, t)) \). If \( p^* \) is maximizing for the standard time constrained problem, then \( (p^* - MC)/p^* = 1/(\eta(p^*, t)) \) where \( \eta \) is the price elasticity of demand measured along demand curve \( q = q(p, t) \). It follows that:

\[
  \frac{p^* - AC}{p^* - MC} = \frac{\varepsilon(p^*, t)}{\eta(p^*, q)}
\]

By identity \( \xi(p^*, q^*) = -\varepsilon(p^*, t^*)/\eta(p^*, q^*) \). Therefore if \( p^* \) maximizes profit over the sales time \( t^* \) of output \( q^* \), and the pair \( (p^*, q^*) \) is also the solution to the problem of maximizing profit within predetermined time frame \( t = t^* \), it will also be optimal to hold price \( p^* \) constant for \( t^* \) units of time since the first order condition resulting from the price constrained optimization problem is satisfied. Thus the endogenously defined period of length \( t^* \) has the characteristics of a unit period: it is a time frame within which the profit made from the sale of output at a constant price is defined and maximized. If \( q^* \) is target inventory, and \( p^* \) is its selling price, then the inventory-to-sales ratio is the endogenously determined length of the unit period. The empirical evidence gathered using automobile industry data in the United States, meat and poultry wholesale data in the United States, and data for Japan's manufacturing sector, does indeed suggest that this interpretation of price, inventory and inventory-to-sales ratios is compatible with the theoretical construct outlined in this essay.

References


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